Why protective measurement establishes the reality of the wave function

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Abstract

It has been debated whether protective measurement implies the reality of the wave function. In this paper, I present a new analysis of the relationship between protective measurement and the reality of the wave function. First, I briefly introduce protective measurements and the ontological models framework for them. Second, I give a simple proof of Hardy's theorem in terms of protective measurements. It shows that when assuming the ontic state of the protected system keeps unchanged during a protective measurement, the wave function must be real. Third, I analyze two suggested ψ-epistemic models of a protective measurement, in which the ontic state of the system is affected by the measurement. It is shown that although these models can explain the appearance of expectation values of observables in a single measurement, their predictions about the variance of the result of a non-ideal protective measurement are different from those of quantum mechanics. Finally, I argue that no ψ-epistemic models exist for an ideal protective measurement in the ontological models framework, and in order to account for the definite result of an ideal protective measurement, the wave function must be a property of the protected system, defined either at a precise instant or during an infinitesimal time interval around an instant. Moreover, this result can also be extended to the wave function of an unprotected system. This new proof of the reality of the wave function does not rely on auxiliary assumptions, and it may help settle the issue about the nature of the wave function.

1 Introduction

The reality of the wave function has been a hot topic of debate since the early days of quantum mechanics. Recent years have witnessed a growing interest in this long-standing question (see, e.g. Pusey, Barrett and Rudolph 2012, Leifer 2014, Gao 2017). Is the wave function ontic, directly representing a state of reality, or epistemic, merely representing a state of incomplete knowledge? Although there are already several important ψ-ontology theorems, a definite answer to this question is still unavailable.

On the one hand, auxiliary assumptions are required to prove the existing ψ-ontology theorems, e.g. the preparation independence assumption for the Pusey-Barrett-Rudolph theorem (Pusey, Barrett and Rudolph, 2012), the ontic indifference assumption for Hardy's theorem (Hardy, 2013), and the parameter independence assumption for the Colbeck-Renner theorem (Colbeck and Renner 2012, 2017). It thus seems impossible to completely rule out the ψ-epistemic view without auxiliary assumptions. Indeed, by removing these auxiliary assumptions, explicit ψ-epistemic (ontological) models can be constructed to reproduce the statistics of quantum mechanics for projective measurements in orthonormal bases in Hilbert spaces of any dimension (Lewis et al, 2012; Aaronson et al, 2013). However, these models do not reproduce the quantum predictions for all possible measurements such as POVMs. As Leifer (2014) rightly pointed out, "it is still possible that there are no ψ-epistemic models that reproduce the quantum predictions for all POVMs, and it may be possible to prove this without auxiliary assumptions."

On the other hand, it has been known that there are other types of quantum measurements besides the conventional projective measurements, such as weak measurements and protective measurements (Aharonov and Vaidman, 1993; Aharonov, Anandan and Vaidman, 1993; Piacentini et al, 2017). Moreover, it has been conjectured that protective measurements, which can measure the expectation values of observables and even the wave function on a single quantum system, may imply the reality of the wave function (Aharonov and Vaidman, 1993; Aharonov, Anandan and Vaidman, 1993, 1996; Gao, 2014, 2015, 2017; Hetzroni and Rohrlich, 2014). However, it has also been argued that this may be not the case (Unruh, 1994; Rovelli, 1994; Dass and Qureshi, 1999; Schlosshauer and Claringbold, 2014; Combes et al, 2018). Thus it is still debatable whether protective measurement really implies the reality of the wave function.

In this paper, I will present a new analysis of the relationship between protective measurement and the reality of the wave function. In particular, I will give a new proof of the reality of the wave function in terms of protective measurements in the ontological models framework. The proof does not rely on auxiliary assumptions such as the preparation independence assumption for the Pusey-Barrett-Rudolph theorem.

The rest of this paper is organized as follows. In Section 2, I first give a brief introduction to protective measurement (PM). It is emphasized that PM is a natural result of the Schrödinger equation; when the wave function of the measured system is protected to be unchanged during a standard von Neumann measurement of an observable, the result is naturally the expectation value of the observable in the wave function of the measured system. Besides, I also briefly introduce two known schemes of PM: the adiabatic-type PM or A-PM and the Zeno-type PM or Z-PM. In Section 3, I then introduce the ontological models framework, which provides a general and rigorous approach to determine whether the wave function is ontic or epistemic. In particular, I introduce the important assumption of the framework for PMs, namely the rule of connecting the underlying ontic states with the results of PMs, which says that the definite result of a PM is determined by the total evolution of the ontic state of the protected system during the PM.

In Section 4, I take Hardy's theorem as an example to show that PM may have implications for the reality of the wave function in the ontological models framework. The key assumption of Hardy's theorem is the ontic indifference assumption, which says that any quantum transformation on a system which leaves unchanged its wave function (including those of PMs) can be performed in such a way that it does not affect the underlying ontic state of the system. I argue that PM provides a simple proof of Hardy's theorem under the ontic indifference assumption. In Section 5, I turn to the dynamics of the ontic state during a PM by analyzing two suggested ψ-epistemic models of a PM (one for a Z-PM and the other for an A-PM), in which the ontic state of the system is affected by the PM. It is shown that although these models can explain the appearance of expectation values of observables in a single measurement, their predictions about the variance of the result of a non-ideal PM are different from those of quantum mechanics.[1]

In Section 6, I further analyze ideal PMs. I argue that no ψ-epistemic models exist for an ideal protective measurement in the ontological models framework, and in order to account for the definite result of an ideal protective measurement, the wave function must be a property of the protected system, defined either at a precise instant or during an infinitesimal time interval around an instant. In Section 7, I extend this result to the wave function of an unprotected system. This proves the reality of the wave function without resorting to auxiliary assumptions. Conclusions are given in the last section.

2 Protective measurements

Protective measurement (PM) is a method to measure the expectation value of an observable on a single quantum system (Aharonov and Vaidman 1993; Aharonov, Anandan and Vaidman 1993; Vaidman 2009). For a conventional projective measurement, the wave function of the measured system

[1]In this paper, when I say a PM I usually mean an ideal PM which yields a definite result unless stated otherwise. Sometimes I also say ideal PM, and this is emphasis.

is in general changed greatly during the measurement, and one obtains an eigenvalue of the measured observable randomly, and the expectation value of the observable can be obtained only as the statistical average of eigenvalues for an ensemble of identically prepared systems. By contrast, during a PM the wave function of the measured system is protected by an appropriate procedure so that it keeps unchanged during the measurement. Then, by the Schrödinger evolution, the measurement result will be directly the expectation value of the measured observable, even if the system is initially not in an eigenstate of the observable.

This result can be seen clearly by the following simple derivation. As for a projective measurement, the interaction Hamiltonian for measuring an observable A is given by the usual form $H_I = g(t)PA$, where $g(t)$ is the time-dependent coupling strength of the interaction, which is a smooth function normalized to $\int_0^T g(t)dt = 1$ during the measurement interval T, and $g(0) = g(T) = 0$, and P is the conjugate momentum of the pointer variable X. When the wave function of the measured system is protected to keep unchanged during the measurement, the evolution of the wave function of the combined system is

$$|\psi(0)\rangle\,|\phi(0)\rangle \rightarrow |\psi(t)\rangle\,|\phi(t)\rangle\,, t > 0, \tag{1}$$

where $|\phi(0)\rangle$ and $|\phi(t)\rangle$ are the wave functions of the measuring device at instants 0 and t, respectively, $|\psi(0)\rangle$ and $|\psi(t)\rangle$ are the wave functions of the measured system at instants 0 and t, respectively, and $|\psi(t)\rangle$ is the same as $|\psi(0)\rangle$ up to an overall phase during the measurement interval $[0, T]$. Then we have

$$\begin{aligned} \frac{d}{dt}\langle\psi(t)\phi(t)|X\,|\psi(t)\phi(t)\rangle &= \frac{1}{i\hbar}\langle\psi(t)\phi(t)|[X, H_I]\,|\psi(t)\phi(t)\rangle \\ &= g(t)\langle\psi(0)|A\,|\psi(0)\rangle\,, \end{aligned} \tag{2}$$

Note that the momentum expectation value of the pointer is zero at the initial instant and the free evolution of the pointer conserves it. This further leads to

$$\langle\phi(T)|X\,|\phi(T)\rangle - \langle\phi(0)|X\,|\phi(0)\rangle = \langle\psi(0)|A\,|\psi(0)\rangle\,, \tag{3}$$

which means that the shift of the center of the pointer wave packet is the expectation value of A in the initial wave function of the measured system. This clearly demonstrates that the result of a measurement of an observable on a system, which does not change the wave function of the system, is the expectation value of the measured observable in the wave function of the measured system.

Since the wave function can be reconstructed from the expectation values of a sufficient number of observables, the wave function of a single quantum system can be measured by a series of PMs. Let the explicit form of the measured wave function at a given instant t be $\psi(x)$, and the measured observable A be (normalized) projection operators on small spatial regions V_n having volume v_n:

$$A = \begin{cases} \frac{1}{v_n}, & \text{if } x \in V_n, \\ 0, & \text{if } x \notin V_n. \end{cases} \tag{4}$$

A PM of A then yields

$$\langle A \rangle = \frac{1}{v_n} \int_{V_n} |\psi(x)|^2 dv, \tag{5}$$

which is the average of the density $\rho(x) = |\psi(x)|^2$ over the small region V_n. Similarly, we can measure another observable $B = \frac{\hbar}{2mi}(A\nabla + \nabla A)$. The measurement yields

$$\langle B \rangle = \frac{1}{v_n} \int_{V_n} \frac{\hbar}{2mi}(\psi^* \nabla \psi - \psi \nabla \psi^*) dv = \frac{1}{v_n} \int_{V_n} j(x) dv. \tag{6}$$

This is the average value of the flux density $j(x)$ in the region V_n. Then when $v_n \to 0$ and after performing measurements in sufficiently many regions V_n we can measure $\rho(x)$ and $j(x)$ everywhere in space. Since the wave function $\psi(x)$ can be uniquely expressed by $\rho(x)$ and $j(x)$ (except for an overall phase factor), the whole wave function of the measured system at a given instant can be measured by PMs.

There are two known schemes of PM (Aharonov and Vaidman 1993; Aharonov, Anandan and Vaidman 1993). The first scheme is to introduce a protective potential such that the wave function of the measured system at a given instant, $|\psi\rangle$, is a nondegenerate energy eigenstate of the total Hamiltonian of the system with finite gap to neighboring energy eigenstates. By this scheme, the measurement of an observable is required to be weak and adiabatic. We may call this scheme the adiabatic-type PM or A-PM. An ideal A-PM requires $T\Delta E \to \infty$, where T is the measurement time, and ΔE is the smallest of the energy differences between $|\psi\rangle$ and other energy eigenstates. The second scheme is via the quantum Zeno effect, and it may be called the Zeno-type PM or Z-PM. The Zeno effect is realized by making frequent projective measurements of an observable, of which the wave function of the measured system at a given instant, $|\psi\rangle$, is a nondegenerate eigenstate. By this scheme, the measurement of the measured observable is not necessarily weak but weaker than the Zeno projective measurements. An ideal Z-PM requires $N \to \infty$, where N is the times of the Zeno projective measurements.

Since the wave function can be measured from a single system by a series of PMs, it seems natural to conjecture that the wave function refers directly to the physical state of the system. In order to investigate whether this conjecture is true, heuristic arguments are not enough, and we need a rigorous approach.

3 Ontological models framework

A general and rigorous approach to determine whether the wave function is ontic or epistemic is the ontological models framework (Spekkens 2005; Harrigan and Spekkens 2010; Leifer 2014). It has three fundamental assumptions.

The first assumption is about the existence of the underlying state of reality. It says that if a physical system is prepared such that quantum mechanics assigns a pure state or wave function to it, then after preparation the system has a well-defined set of physical properties or an underlying ontic state, which is usually represented by a mathematical object, λ. Here a strict ψ-ontic/epistemic distinction can be made. In a ψ-ontic (ontological) model, the ontic state of a physical system uniquely determines its wave function, and thus the wave function is a property of the system. While in a ψ-epistemic (ontological) model, there are at least two wave functions which are compatible with the same ontic state of a physical system. In this case, the wave function represents a state of incomplete knowledge – an epistemic state – about the actual ontic state of the system. In general, the wave function corresponds to a probability distribution $p(\lambda|P)$ over all possible ontic states when the preparation is P, and the probability distributions corresponding to two different wave functions may overlap.

The second assumption of the ontological models framework is about the dynamics of the ontic state. It says that a unitary transformation U on a wave function is represented in general by a stochastic transformation on the ontic state space. For example, for a finite ontic state space and the stochastic transformation being a Markov kernel γ, $\gamma_\lambda(\lambda')$ is the probability that the dynamics causes λ to make a transition to λ', both of which are on the ontic state space (Leifer 2014).

In order to investigate whether an ontological model is consistent with the empirical predictions of quantum mechanics, we also need a rule of connecting the underlying ontic states with the results of measurements. This is the third assumption of the ontological models framework, which says that when a measurement is performed, the behaviour of the measuring device is determined only by the ontic state of the system, along with the physical properties of the measuring device. For a projective measurement M, this assumption means that the ontic state λ of a physical system determines the probability $p(k|\lambda, M)$ of different results k for the measurement M on the system. The consistency with the predictions of quantum mechanics then requires the following relation: $\int d\lambda p(k|\lambda, M)p(\lambda|P) = p(k|M, P)$, where $p(k|M, P)$ is the Born probability of k given M and P.

For a PM, which yields a definite measurement result, it seems that the above assumption should mean that the ontic state of a physical system determines the definite result of the PM on the system (Gao 2015). However, this view is debatable. Unlike a projective measurement, a PM such as an A-PM may take a very long time, and thus it seems not reasonable to assume that when a PM is performed, the behaviour of the measuring device is determined by the ontic state of the measured system (along with the physical properties of the measuring device) immediately before the PM, whether the ontic state of the measured system is affected or not during the PM. A more reasonable assumption for PMs is that the ontic state of the measured system may be affected (by both the protection procedure and the measuring device) and thus evolve in a certain way during a PM, and the definite result of the PM is determined by the total evolution of the ontic state of the system during the PM, not simply by the initial ontic state of the system (see also Gao 2017).

In the following sections, I will analyze whether PM has implications for the reality of the wave function in the above ontological models framework. I will first give a very simple proof of Hardy's theorem in terms of PMs, which suggests that the answer may be yes.

4 A simple proof of Hardy's theorem

Hardy's theorem is one of the three important ψ-ontology theorems appeared in recent years (Hardy 2013). It is based on three assumptions. The first one is realism, which says that each time a system is prepared there exists an underlying state of reality or an ontic state, denoted by λ. This is just the first assumption of the ontological models framework. The second assumption of Hardy's theorem is possibilistic completeness, which says that the ontic state, λ, is sufficient to determine whether any outcome of any (projective) measurement has probability equal to zero of occurring or not. This is a weaker version of the second assumption of the ontological models framework, according to which the ontic state determines the probabilities for the results of projective measurements. The third assumption of Hardy's theorem is an auxiliary assumption and also the key assumption of the theorem, called ontic indifference, which says that any quantum transformation on a system which leaves unchanged any given wave function $|\psi\rangle$ can be performed in such a way that it does not affect every underlying ontic state which is assigned a nonzero probability by $|\psi\rangle$.

Hardy's theorem can be illustrated with a simple example (Leifer 2014). Assume there are two nonorthogonal states $|\psi_1\rangle$ and $\frac{1}{\sqrt{2}}(|\psi_1\rangle + |\psi_2\rangle)$, which are compatible with the same ontic state λ as required by the ψ-epistemic view. Consider a unitary evolution which leaves $|\psi_1\rangle$ invariant but changes $\frac{1}{\sqrt{2}}(|\psi_1\rangle + |\psi_2\rangle)$ to its orthogonal state $\frac{1}{\sqrt{2}}(|\psi_1\rangle - |\psi_2\rangle)$. Since two orthogonal states correspond to different ontic states,[2] the original ontic state λ must be changed by the unitary evolution. Then if the unitary evolution that leaves $|\psi_1\rangle$ invariant also leaves the underlying ontic state λ invariant as the ontic indifference assumption requires,[3] there will be a contradiction. In other words, under the above three assumptions we can prove that the two nonorthogonal state $|\psi_1\rangle$ and $\frac{1}{\sqrt{2}}(|\psi_1\rangle + |\psi_2\rangle)$ are ontologically distinct.

This is the simplest example of Hardy's theorem. A complete proof of this theorem requires a more complex mathematical analysis. In particular, the proof requires that the Hilbert space associated with the system in question is infinite dimensional. If this dimension, $N + 1$, is finite, then it can be proved only that any pair of wave functions, $|\psi\rangle$ and $|\phi\rangle$, for which $|\langle\phi|\psi\rangle|^2 \geq \frac{N-1}{N}$ must have non-overlapping distributions over the ontic states (under the above three assumptions). Interestingly and surprisingly, however, even if the ontic indifference assumption holds only for a single wave function, Hardy's theorem can also be proved (Hardy, 2013; Patra, Pironio and Massar, 2013).

In the following, I will show that under the key assumption of Hardy's theorem, namely the ontic indifference assumption, protective measurement implies the reality of the wave function in the ontological models framework. This will provide a simple proof of Hardy's theorem.

For a PM, the wave function of the protected system keeps unchanged during the measurement. Then, according to the ontic indifference assumption, the ontic state of the system is not changed either during the measurement. In the ontological models framework for PMs, it is assumed that the total evolution of the ontic state of the protected system during a PM determines the definite result of the PM (this is the third assumption of the framework). Then, since the ontic state of the system does not change during the PM, the initial ontic state of the system already determines the definite result of the PM, namely the expectation value of the measured observable. This means that the expectation value of each measured observable is a property of the protected system. Since a wave function can be constructed from the expectation values of a sufficient number of observables, the wave function of the protected system is also a property of the system.

In order to extend this result to the wave function of an unprotected system, we may reuse the ontic indifference assumption. Since a protection procedure such as that in an A-PM is a unitary transformation which does not change the wave function of the measured system, the ontic indifference assumption requires that each ontic state of the system (which is assigned

[2]Note that the possibilistic completeness assumption is needed to prove this result.

[3]One strong motivation for this assumption is locality. When $|\psi_1\rangle$ and $|\psi_2\rangle$ are two spatially separated states prepared in regions 1 and 2 respectively, it seems reasonable to assume that the local evolution of the ontic state in region 2 does not influence the ontic state in region 1.

a nonzero probability by the wave function) is not changed after the transformation. Then, if the wave function is a part of the ontic state of the protected system, it is also a part of the ontic state of the original, unprotected system. In other words, the wave function of an unprotected system is also real.

Hardy's theorem can also be proved under the restricted ontic indifference assumption, namely the theorem can be proved even if the ontic indifference assumption holds only for a single wave function (Hardy, 2013; Patra, Pironio and Massar, 2013). It seems that the above proof in terms of PMs cannot go through if the ontic indifference assumption holds only for a single wave function; in this case, the proof will only show that this wave function is real. However, if one considers the fact that the scheme of PM is the same for all wave functions, then it is arguable that if the ontic indifference assumption holds for a single wave function, it will also hold for other wave functions. In this sense, the above proof in terms of PMs may also go through under the restricted ontic indifference assumption.

5 On two ψ-epistemic models of PMs

The above analysis shows that when assuming the ontic state of the protected system keeps unchanged during a PM, the wave function must be real. Then, an ψ-epistemic model must assume that the ontic state of the protected system evolves in a certain way in order to account for PMs. Concretely speaking, the ontic state of the protected system must undergo a dynamical process to generate the result of the PM, which is the expectation value of the measured observable. The question is: can any dynamics of the ontic state account for PMs? In this section, I will analyze two recently suggested ψ-epistemic models of PMs, one for Z-PMs and the other for A-PMs (Combes et al 2018).

For a Z-PM, there is an ensemble of identically copies of the measured system, which is prepared by the protection procedure, namely the frequent Zeno projective measurements, when the protection is successful. Thus, it seems possible that the result of the Z-PM, namely the expectation value of the measured observable, is also obtained as the ensemble average of the eigenvalues of the measured observable as for conventional projective measurements. Indeed, Combes et al (2018) suggested such an ψ-epistemic model for a Z-PM.[4] The model assumes that any observable A of the measured system has a definite value at any time, which is one of the eigenvalues of A. Similarly, the pointer of the measuring device also has a definite position at any time, which is the same as the measured position predicted by quantum mechanics. When each Zeno projective measurement results in the wave function of the measured system being in $|\psi\rangle$, it also random-

[4]The model discussed below is an extension of the original model for a spin-1/2 particle.

izes the value of A and make it be a_i with probability p_i, where a_i is an eigenvalue of A, and $p_i = |\langle a_i|\psi\rangle|^2$ is the corresponding Born probability. Then the measured system shifts the pointer by a_i/N after the follow-up measurement of A. In the end, the total pointer shift, denoted by Δx, will be the expectation value of A when N approaches infinity:

$$\Delta x = \lim_{N\to\infty} \sum_i n_i a_i/N = \sum_i p_i a_i = \langle A\rangle. \tag{7}$$

This ψ-epistemic model shows that the result of a Z-PM, the expectation value of the measured observable, may be generated from the eigenvalues of the observable for an ensemble of identically copies of the measured system, which is prepared by the protection procedure in the Z-PM. However, as Combes et al (2018) also pointed out, the model does not aim to provide a complete account of a Z-PM, which means that the predictions of the model may be not fully consistent with those of quantum mechanics. This is indeed the case, since it can be shown that this ψ-epistemic model and quantum mechanics have different predictions about the variance of the result of a Z-PM with finite N.

A Z-PM is composed of N identical units, each of which contains a protecting system and a measuring system. In the above ψ-epistemic model, the pointer shift generated by the i-th Z-PM unit, Δx_i, has a probability distribution

$$p(\Delta x_i = a_k) = |\langle a_k|\psi\rangle|^2. \tag{8}$$

Thus we have $Var(\Delta x_i) = Var(A)/N^2$ for any i, where $Var(\cdot)$ is the variance, and $Var(A) \equiv \langle A^2\rangle - \langle A\rangle^2$. Then the variance of the final position of the pointer after the Z-PM is

$$Var(x_f) = Var(x_0 + \sum_i \Delta x_i), \tag{9}$$

where x_f is the final position of the pointer, and x_0 is the initial position of the pointer. Since each random process Δx_i is independent with each other and also independent of the initial position of the pointer in the model, we have

$$Var(x_f) = Var(x_0) + Var(\sum_i \Delta x_i) = Var(x_0) + \frac{Var(A)}{N}. \tag{10}$$

On the other hand, according to quantum mechanics, the branch of the state of the combined system after the Z-PM (i.e. after N such measurements), in which each Zeno projective measurement results in the state of the measured system being in $|\psi\rangle$, is (up to the first order of 1/N)

$$\begin{aligned} |t=T\rangle &= |\psi\rangle\,|\phi(x_0+\langle A\rangle)\rangle \\ &+\frac{Var(A)}{2N}\,|\psi\rangle\,|\phi''(x_0+\langle A\rangle)\rangle\,, \end{aligned} \tag{11}$$

where $\phi(x_0)$ is the initial pointer wave packet. Suppose the initial pointer wavepacket is a Gaussian wavepacket. Then we can calculate the variance of the final measuerd position of the pointer, which is

$$Var(x_f) = Var(x_0) + \frac{Var(A)}{N} Var(x_0)(k_1 + k_2 Var(x_0)), \tag{12}$$

where $Var(x_0)$ is the variance of the initial measured position of the pointer, and k_1, k_2 are numerical constants related to the Gaussian wavepacket.

It can be seen that the above ψ-epistemic model and quantum mechanics give obviously different predictions about the variance of the result of a Z-PM with finite N. In the model, the first order term does not depend on the initial position variance of the pointer, but in quantum mechanics it does. Certainly, one may revise the above ψ-epistemic model so that its predictions may be consistent with those of quantum mechanics for the first order of $1/N$. But it seems extremely difficult or even impossible to revise the model so that its predictions are consistent with those of quantum mechanics for all orders of N, except that the dynamics of the ontic state in the ψ-epistemic model is also the Schrödinger equation for the wave function.[5] But then the wave function will be a part of the ontic state of the system, and the model will be ψ-ontic, not ψ-epistemic.

Combes et al (2018) also proposed an ψ-epistemic model for an A-PM for some observables. In the model, the wave function is a coherent state of a quantum harmonic oscillator. The Hamiltonian of the system is set to make this state be its nondegenerate ground state. Then the system is coupled to a pointer via the usual interaction Hamiltonian $H_I = PA/T$ for a time duration T, where P is the conjugate momentum of the pointer variable X, and A is a measured quadrature observable.[6] In the Heisenberg picture, the pointer variable at time t during the A-PM is (up to the first order of 1/T)

$$X(t) = X(0) + \frac{t}{T}\langle A\rangle + \frac{1}{T}[q(0)\sin t + p(0)(1-\cos t)], \tag{13}$$

where $q(0)$ is the initial position of the system, and $p(0)$ is the initial momentum of the system.

In this ψ-epistemic model for an A-PM, as in the previous ψ-epistemic model for a Z-PM, it is still assumed that any observable A of a system has a definite value at any time, which is one of the eigenvalues of A, and

[5] I will discuss this point in more detail later.

[6] Here I use a notation somewhat different from the orginal one.

in particular, the pointer also has a definite position at any time, which is the same as the measured position predicted by quantum mechanics. Then, when $T \to \infty$, we have $X(T) = X(0) + \langle A \rangle$, which means that the pointer shift is indeed the result of the A-PM, namely the expectation value of the measured observable.

However, it can be seen that like the previous ψ-epistemic model for a Z-PM, this ψ-epistemic model for an A-PM is also inconsistent with quantum mechanics in the predictions about the variance of the measurement result for non-ideal situations in which the measurement time T in finite. According to the model, the variance of the final position of the pointer after the A-PM is

$$Var(x_f) = Var(x_0) + \frac{1}{T^2}[Var(q_0)\sin^2 T + Var(p_0)(1-\cos T)^2], \quad (14)$$

where $Var(q_0)$ is the initial position variance of the system, and $Var(p_0)$ is the initial momentum variance of the system. This time the discrepancy is more obvious. Quantum mechanics predicts that the variance of the final measured position of the pointer after the A-PM should have the first order term which depends on the initial measured position of the pointer, while the above model predicts that there is no such a term at all.[7]

Again, one may think that the above ψ-epistemic model for an A-PM can be revised so that its predictions are consistent with those of quantum

[7]In Combes et al (2018), the authors claimed that for an A-PM, all of the information about the expectation value of the measured observable obtained by the measurement comes from the protection operation (i.e. the protection Hamiltonian) rather than from the system itself. But this claim is not proved. In the above model, the authors said that the expectation value of the measured observable is a parameter in the Hamiltonian (see Eq. (25) of the paper). But this is arguably a mathematical trick. It just rewrites the usual interaction Hamiltonian $H_I = g(t)PA$ as $H_I = g(t)P(A' + \langle A \rangle)$, where $A' = A - \langle A \rangle$. Note that even if what the authors said is true, the expectation value is not a parameter in the protection Hamiltonian, but a parameter in the interaction Hamiltonian. Moreover, it is obvious that the expectation value of the measured observable depends on the measured observable and the wave function of the measured system in mathematics. In the model, the expectation value of the measured observable is $c_\theta = c_q \cos\theta + c_p \sin\theta$, where c_q and c_p come from the wave function of the system (see Eq. (23) of the paper), and $\cos\theta$ and $\sin\theta$ come from the measured observable. In fact, the above claim cannot be true. The reason is that the same protection operation for an A-PM can protect infinitely many energy eiegnstates, while the expectation values of the measured observable in these states are different in general, and thus the system needs to pick out one of these states as the one that it is in at least. The authors admitted this point for a Z-PM. In the final analysis, we need an analysis of the physical mechanism of a PM, such as the suggested ψ-epistemic models, in order to answer the question of where the information obtained by the measurement comes from. In mathematics, the information about the wave function of the measured system may be indeed present in the POVM for a Z-PM or in the Hamiltonian for an A-PM. But this does not imply that the information obtained by a PM must come from the protection operation rather than from the system itself, since it is also present in the wave function of the system more obviously, and it may directly come from the system itself if the wave function is a property of the system.

mechanics for the first order of $1/T$. However, it seems impossible to obtain the consistency, let alone the consistency for all orders of $1/T$. Just look at the final wave function of the combining system after an A-PM, which is (up to the first order of $1/T$)

$$|t=T\rangle = |\psi\rangle\,|\phi(x_0+\langle A\rangle)\rangle + \frac{1}{T}\sum_m \frac{1}{E-E_m}\,|E_m\rangle$$
$$\times\,[\langle E_m|A\,|\psi\rangle \left|\widetilde{\phi}(x_0+\langle A\rangle)\right\rangle - \mathrm{e}^{\frac{i}{\hbar}(E-E_m)T}\langle\psi|A\,|E_m\rangle \left|\widetilde{\phi}(x_0+\langle A\rangle_m)\right\rangle], \quad (15)$$

where E is the energy of the measured state $|\psi\rangle$, $|E_m\rangle$ are the other energy eigenstates, E_m are the corresponding energy eigenvalues, $\left|\widetilde{\phi}(x_0)\right\rangle$ is a distorted version of the initial pointer wave packet (see Schlosshauer and Claringbold 2014), and $\langle A\rangle_m \equiv \langle E_m|A\,|E_m\rangle$. Since energy, unlike the wave function, is still a property of a single system in an ψ-epistemic model, the dynamics of the ontic state of the system in the model must generate the infinitely many energy eigenvalues, $\{E_m\}$, during each measurement in order to make the same predictions with quantum mechanics about the variance of the result for the first order of $1/T$. While in order to generate exactly the same infinitely many energy eigenvalues which are derived from the Schrödinger equation (e.g. the energy levels for an infinite square well potential), it seems that the dynamics of the ontic state must be also the Schrödinger equation. But then the wave function will be a part of the ontic state of the system, and the model will be ψ-ontic, not ψ-epistemic.

Although the above ψ-epistemic models for Z-PMs and A-PMs are not fully consistent with quantum mechanics for finite N and T, it seems that the consistency may be reached for ideal situations, namely when $N\to\infty$ and $T\to\infty$; in this case, they give the same predictions about the variance of the result. In the next section, however, I will argue that no ψ-epistemic models exist for ideal PMs.

6 The wave function of a protected system is real

In the following, I will argue that PM may imply the reality of the wave function of a protected system in the ontological models framework.

Consider an ideal PM of an observable A. The initial wave function of the measured system is $|\psi\rangle$. As before, the interaction Hamiltonian is given by the usual form $H_I = g(t)PA$, where $g(t)$ is the time-dependent coupling strength of the interaction, which is a smooth function normalized to $\int_0^T g(t)dt = 1$ during the measurement interval T, and $g(0)=g(T)=0$, and P is the conjugate momentum of the pointer variable X. Then the pointer shift after a time δt during the PM is:

$$\Delta x = \langle A \rangle \int_0^{\delta t} g(t)dt, \tag{16}$$

where $\Delta x = \langle X \rangle_{\delta t} - \langle X \rangle_0$, $\langle X \rangle_0$ is the center of the initial pointer wavepacket, $\langle X \rangle_{\delta t}$ is the center of the pointer wavepacket after δt, and $\langle A \rangle$ is the expectation value of the measured observable A. Here I used the fact that the wave function of the measured system is not changed during the PM.

When $\delta t = T$ we obtain $\Delta x = \langle A \rangle$, namely the result of the PM is the expectation value of the measured observable. According to the third assumption of the ontological models framework for PMs, the result of the PM, $\langle A \rangle$, is determined by the total evolution of the ontic state of the system during the measurement interval T. This means that $\langle A \rangle$ is an average property of the protected system during the measurement interval T. This is already an interesting result (see also Aharonov, Anandan and Vaidman, 1996).

Furthermore, when the time-dependent coupling strength $g(t)$ is known, we can also obtain the result $\langle A \rangle$ after any $\delta t > 0$ during the PM (see below for a more detailed discussion). Then, $\langle A \rangle$ is also an average property of the protected system during the time interval δt. Since δt can be arbitrarily small, this means that $\langle A \rangle$ is actually a property of the protected system defined during an infinitesimal time interval around the initial instant $t = 0$. Moreover, since a wave function can be constructed from the expectation values of a sufficient number of observables, the initial wave function of the protected system, $|\psi\rangle$, is also a property of the system defined during an infinitesimal time interval around the initial instant.

Here it is worth noting that the ontic state of a physical system can be defined either at a precise instant or during an infinitesimal time interval around an instant. The former is like the definition of position in classical mechanics, and the latter is like the definition of velocity in classical mechanics. Thus, according to the above analysis, the wave function of a protected system is a part of the ontic state of the system. If the evolution of the ontic state is continuous, then the wave function will be an instantaneous property of the system. While if the evolution of the ontic state is discontinuous, then the wave function will be a property of the system defined during an infinitesimal time interval around a given instant. In this case, the wave function cannot be the complete ontic state of the system; rather, the ontic state of the system will include both the wave function and the instantaneous properties of the system which evolve in a discontinuous way.

There is also one point which needs to be clarified. It is about the understanding of the arbitrary smallness of δt in Eq. (16). In order to ensure that the result of a PM after δt can be read out, it is required that

$\langle A \rangle \int_0^{\delta t} g(t)dt > W_{\delta t}$, where $W_{\delta t} = \sqrt{\frac{1}{2}(W_0^2 + \frac{\delta t^2}{M^2 W_0^2})}$ is the width of the pointer wave packet after δt, W_0 is the initial width of the pointer wave packet, and M is the mass of the pointer.[8] The spread of the pointer wave packet is smaller than its initial width when the mass of the pointer is large enough, namely $M > \delta t / W_0^2$. While there is a restriction on the initial width of the pointer wave packet in order that the scheme of a PM is valid. Roughly speaking, for a Z-PM, the restriction is $W_0 > \langle A \rangle \int_0^{T/N} g(t)dt$, and for an A-PM, the restriction is $W_0 > \langle A \rangle \int_0^{\hbar/\Delta E} g(t)dt$, where ΔE is the smallest of the energy differences between $|\psi\rangle$ and other energy eigenstates. Then, the above requirement is equivalent to $\delta t > T/N$ for a Z-PM or $\delta t > \hbar/\Delta E$ for an A-PM. This means that when $N \to \infty$ for a Z-PM or $\Delta E \to \infty$ for an A-PM, the requirement can be satisfied for any $\delta t > 0$, or in other words, the result of a PM can be read out after any $\delta t > 0$.

Here one may object that for a Z-PM, no matter how small δt is, if only the result, namely the expectation value of the measured observable, can be read out, there may alway exist an ψ-epistemic model that can account for the result, since the condition for reading out the result is $\delta t > T/N$, depending on the frequent Zeno projective measurements, and in this case there may exist a dynamical process that can generate the result, like the one discussed in the last section. Agreed; this may be indeed true for any finite N, although a complete ψ-epistemic model is still unavailable for a Z-PM with finite N. But the key point is that for an ideal Z-PM, namely when $N \to \infty$, the dynamical process that generates the expectation value of the measured observable, if it does exist, will happen during an infinitesimal time interval and thus it will be a part of the ontic state of the system, which is defined during an infinitesimal time interval around a given instant. In other words, when $N \to \infty$, the ψ-epistemic model becomes an ψ-ontic model. Note that the above argument does not claim that an ψ-epistemic model cannot account for the result of a non-ideal Z-PM; rather, it only concludes that an ψ-epistemic model cannot account for an ideal Z-PM. This is enough to prove the reality of the wave function of a protected system.

For an A-PM, the situation seems somewhat different and better. The condition for reading out the result of an A-PM is $\delta t > \hbar/\Delta E$, and it is arguable that the dynamical process that may generate the result is independent of ΔE, which is also determined by the enegy of another energy eigenstate.[9] Then, given any dynamical process that takes a finite time (if

[8]Here it is also worth noting that for a non-ideal PM, there is also a contribution to $W_{\delta t}$ from the measuring interaction. But the contribution is as small as $O(1/N^2)$ for a Z-PM or $O(1/T^2)$ for an A-PM, while the pointer shift is in the order of $1/N$ for a Z-PM or $1/T$ for an A-PM.

[9]One may argue that the dynamical process of the protected system may also depend on the energies of other energy eigenstates, since the protection is not perfect and the wave function of the protected system is also disturbed. However, it is arguable that the

it took only an infinitesimal time, then the wave function would be already a property of the protected system), we can always adjust ΔE so that the result of an A-PM can be read out before it is generated by the dynamical process. This means that an ψ-epistemic model cannot account for the A-PM. Since this argument does not rely on the ideal condition $\Delta E \to \infty$, it is stronger than the above argument for a Z-PM.

To sum up, I have argued that in order to account for the definite result of an ideal PM, the wave function must be a property of the protected system, defined either at a precise instant or during an infinitesimal time interval around an instant. The argument does not resort to auxiliary assumptions. This also means that any ψ-epistemic model, in which there is at least a wave function which is not a property of a single protected system, cannot account for PMs. In other words, PM implies the reality of the wave function of a protected system.

7 The reality of the wave function

In this section, I will try to extend the above result to the wave function of an unprotected system. I will give three arguments, from the weakest to the strongest.

The first argument is based on an auxiliary assumption similar to the assumption of preparation independence for the Pusey-Barrett-Rudolph theorem. An obvious assumption is preparation noncontextuality, which says that if two preparation procedures are represented by the same wave function, then there should be no difference between them at the ontological level, or in other words, a wave function corresponds to a unique probability distribution of the ontic state.[10] Based on this assumption, we can directly extend the above result to the wave function of an unprotected system. Since the wave function of an unprotected system does not change after the system is protected, the ontic states of the two systems have the same probability distribution according to the preparation noncontextuality assumption. Then the wave function being a part of the ontic state of the protected system implies that the wave function is also a part of the ontic

dependence of the dynamical process on the energies of other energy eigenstates should be weak, being in the order of the disturbance $1/T$. Moreover, as noted before, since the dynamics of the ontic state is not the Schrödinger equation for the wave function, it seems impossible that the new equation of dynamics also gives exactly the same infinitely many energy levels which are derived from the Schrödinger equation.

[10] According to Leifer (2014), "Preparation noncontextuality says that if there is no difference between two preparation procedures in terms of the observable statistics they predict, i.e. they are represented by the same quantum state, then there should be no difference between them at the ontological level either, i.e. they should be represented by the same probability." Due to the existence of PMs, the observable statistics in this definition of preparation noncontextuality should be restricted for projective measurements.

state of the unprotected system. In other words, the wave function of the unprotected system is also real.

Note that the ψ-ontic view implies preparation noncontextuality, and preparation contextuality implies the ψ-epistemic view. Thus it is preparation noncontextuality, not preparation contextuality, that can be assumed in proving the ψ-ontic view or the ψ-epistemic view. So far, the necessity of preparation contextuality has only been established for mixed states. If preparation contextuality is true for one pure state, then the ψ-epistemic view will be proved. On the other hand, the reality of the wave function of a protected system (for A-PM) shows that all energy eigenstates are real, which implies that the preparation noncontextuality assumption is true at least for all energy eigenstates.

We may also use a weaker auxiliary assumption in order to prove the reality of the wave function of an unprotected system. It is not required that a wave function must correspond to a unique probability distribution of the ontic state. It is only required that a wave function corresponds to a unique set of the ontic states which are assigned a zero probability by it. This means that if an ontic state can be prepared by a preparation procedure for a wave functon (with a nonzero probability), then it can also be prepared by other preparation procedures for the wave functon (with a nonzero probability). Then, since the wave function is a part of the ontic state of a protected system, which is prepared by a protection procedure, it can also be prepared by other non-protection preparation procedures, which means that the wave function is also a part of the ontic state of an unprotected system. In other words, the wave function of an unprotected system is also real.[11]

My second argument is based on a meta assumption concerning the nature of the laws of physics, which is different from a usual auxiliary assumption about the ontic state of a system and its dynamics. The key is to notice that the statistics of the projective measurements of all observables are the same for an unprotected system and the corresponding protected system (e.g. which is protected by a potential). Then, if the wave function of the former is epistemic and the wave function of the later is ontic, then we will have two different mechanisms to explain the statistics, as well as other quantum phenomena (except those related to PMs). For example, we will use the Schrödinger equation for the protected system, whose ontic state is the wave function, and use another different equation of motion for the unprotected system, whose ontic state is not the wave function. Thus,

[11]Certainly, one may also prove this result by resorting to other auxiliary assumptions. For example, one may assume that the wave function of any system is not in the ontic state space of the system for the ψ-epistemic view. Then one can derive a contradicton if the wave function of an unprotected system is not real. Note that including the wave function in the ontic state space seems to be against the spirit of the ψ-epistemic view. I will discuss this point in more detail later.

if we assume that there is a unique mechanism or equation of motion to explain the quantum statistics, which is a meta assumption about the laws of physics,[12] then the reality of the wave function of a protected system will imply the reality of the wave function of an unprotected system.[13]

The above two arguments are both based on an auxiliary assumption. In order to prove the reality of the wave function of an unprotected system without resorting to auxiliary assumptions, we need a further analysis of the change of the ontic state after the protection procedure, such as the protection potential for an A-PM, is added or removed.[14] This leads to my third argument. Assume that a wave function of an unprotected system, $|\psi\rangle$, is not real and corresponds to a probability distribution $p(\lambda|P)$ over all possible ontic states λ when the preparation is P. Then after the protection procedure is added, it is required that all these ontic states λ should change to the same ontic state $|\psi\rangle$.[15] This seems possible since the protection procedure relates to the wave function. On the other hand, when the protection procedure is removed, it is required that the ontic state $|\psi\rangle$ should change back to the ontic state λ with a probability $p(\lambda)$ compatible with the wave function of the unprotected system (which is not necessarily the same as the original probability $p(\lambda|P)$ when preparation contextuality is allowed). The question is: can *this* requirement be satisfied?

The answer is arguably negative. An essential reason is as follows. When the protection procedure is added, the change of the ontic state from any possible λ to $|\psi\rangle$ is a deterministic process. While when the protection procedure is removed, the reversal change of the ontic state from $|\psi\rangle$ to any possible λ is a random process; $|\psi\rangle$ changes to λ with a probability $p(\lambda)$. But no physical mechanism can make the reversal of a deterministic process be a random process. Note that this problem can be avoided when the wave function of an unprotected system, like the wave function of a protected system, is also real. In this case, both processes are deterministic; the ontic state does not change when the protection procedure is added or removed.

It can be seen that preparation contextuality will not weaken but strengthen the above argument. If assuming the wave function of an unprotected system, unlike the wave function of a protected system, is not real, then we

[12]Note that this assumption is not the same as preparation noncontextuality, and the former does not require the latter either. For example, if the ψ-epistemic view is true, in which preparation contextuality is allowed, it may also provide a unique mechanism to explain the quantum statistics.

[13]Here it may be also worth noting that the ψ-epistemic models are proposed with the aim of explaining the quantum phenomena which seem puzzling when assuming the ψ-ontic view. But if these phenomena also need an ψ-ontic explanation, then the aim of the ψ-epistemic models will be misplaced.

[14]I thank Matt Pusey for his helpful comments on an earlier draft of this paper, in which he pointed out the necessity of such an analysis.

[15]Here I suppose the completeness of the ontic state being the wave function for simplicity. This does not influence the argument below.

already assume preparation contextuality; a wave function does not correspond to a unique probability distribution of the ontic state. But if preparation contextuality is allowed for unprotected systems, then the above requirement cannot be satisfied. In this case, the wave function of an unprotected system, $|\psi\rangle$, does not correspond to a unique probability distribution of the ontic state. Then, even if the reversal of a deterministic process can be a random process, and in particular, the ontic state can change from $|\psi\rangle$ to any possible λ randomly, the probability of the change cannot be determined in general.

In fact, even if the wave function of an unprotected system, $|\psi\rangle$, corresponds to a unique probability distribution of the ontic state, $p(\lambda)$, it seems that the above requirement cannot be satisfied either. The reason is that the unitary transformation corresponding to the removal of the protection procedure, as well as the ontic state of the protected system before the removal, $|\psi\rangle$, cannot determine the probability of the change from $|\psi\rangle$ to any possible λ, which is required to be $p(\lambda)$. The protection procedure relates only to $|\psi\rangle$, while $|\psi\rangle$ does not determine $p(\lambda)$. This can be seen from the rule connecting $|\psi\rangle$ with $p(\lambda)$: $\int p(k|\lambda, M)p(\lambda)d\lambda = |\langle\psi|k\rangle|^2$, where $p(k|\lambda, M)$ is the probability of different results k for the measurement M on the system whose ontic state is λ. In mathematics, $p(k|\lambda, M)$ and $p(\lambda)$ determine $|\psi\rangle$, but $|\psi\rangle$ does not determine $p(k|\lambda, M)$ and $p(\lambda)$.

To sum up, I have argued that the wave function of an unprotected system, like the wave function of a protected system, is also real. The argument may not resort to auxiliary assumptions.

8 Conclusion

Since the discovery of the new method of protective measurement in quantum mechanics by Aharonov, Vaidman and Anandan in 1993, it has been debated whether it implies the reality of the wave function. On the one hand, since protective measurement can measure the wave function from a single system, it seems tempting and natural to assume that the wave function is a property of a single system. On the other hand, since protective measurement must involve a protection procedure related to the wave function of the measured system, it seems also possible that the wave function is not a property of the system, but generated by the evolution of the actual ontic state of the system induced by the protection procedure.

In this paper, I present a new analysis of the relationship between protective measurement and the reality of the wave function, and argue that the former may indeed imply the latter in the ontological models framework. I first give a simple proof of Hardy's theorem, which shows that when assuming the ontic state of the protected system keeps unchanged during a protective measurement, the wave function must be real. I then show that although two suggested ψ-epistemic models of a protective measurement can explain the appearance of expectation values of observables in the measurement by adding a certain dynamics of the ontic state, their predictions about the variance of the result of a non-ideal protective measurement are different from those of quantum mechanics. Finally, I argue that no ψ-epistemic models exist for ideal protective measurements in the ontological models framework, and in order to account for the definite result of an ideal protective measurement, the wave function must be a property of the protected system. Moreover, this result can also be extended to the wave functions of unprotected systems.

When considering only conventional projective measurements, auxiliary assumptions are needed to prove the reality of the wave function. For example, the Pusey-Barrett-Rudolph theorem is based on an additional assumption of preparation independence. The new proof in terms of protective measurements does not rely on auxiliary assumptions, and it may help settle the issue about the nature of the wave function.

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